

Reaching the End of Unbiasedness: Uncovering Implicit Limitations of Click-Based Learning to Rank

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Main Question

How far can the prevalent approach in the Unbiased Learning-to-Rank field take us?

In Other Words

What are the limits of the field's current approach to Unbiased Learning-to-Rank?

Introduction: Click-Based Learning-to-Rank



Click-Based Learning-to-Rank:

- Optimization of **ranking models** w.r.t. ranking metrics for **search** or **recommendation** based on **user-interactions** (i.e. clicks).
- Encapsulates unbiased / counterfactual / offline / online LTR.

Core Problem:

- Mismatch between click-probabilities and relevances.
- Other factors beside relevance affect CTR as well.

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Existing Click-Based LTR Approach



Vardasbi et al. (2020) assume the click model, with doc. d, query q, rank k, relevance $R_{d|q}$, bias parameters α_k and β_k per rank:

$$P(C = 1 \mid d, k, q) = \alpha_k R_{d|q} + \beta_k.$$

A. Vardasbi, H. Oosterhuis, and M. de Rijke. When inverse propensity scoring does not work: Affine corrections for unbiased learning to rank. CIKM 2020.



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They propose an estimator based on the inverse transformation:

$$\hat{R}_{d|q} = \frac{1}{N} \sum_{i=1}^{N} \frac{c_i(d) - \hat{\beta}_{k_i(d)}}{\hat{\alpha}_{k_i(d)}}$$

Straightforwardly **unbiased** if bias is correctly estimated:

$$\left(\hat{\alpha} = \alpha \land \hat{\beta} = \beta\right) \longrightarrow \mathbb{E}\left[\hat{R}_{d|q}\right] = R_{d|q}.$$

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A basic click model estimates by optimizing parameters of a predictive model:

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The values of R, α and β are inferred by **minimizing** a **loss** function:

$$\hat{\mathcal{L}} = \frac{-1}{N} \sum_{i=1}^{N} c_i(d) \log \left(\hat{\alpha}_{k_i(d)} \hat{R}_{d|q} + \hat{\beta}_{k_i(d)} \right) + \left(1 - c_i(d) \right) \log \left(1 - \hat{\alpha}_{k_i(d)} \hat{R}_{d|q} - \hat{\beta}_{k_i(d)} \right).$$



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The expected loss is the cross-entropy loss on click probabilities:

$$\mathbb{E}[-\hat{\mathcal{L}}] = P(C = 1 \mid d, q) \log \hat{P}(C = 1 \mid d, q) + P(C = 0 \mid d, q) \log \hat{P}(C = 0 \mid d, q).$$

Methodology: Uncovering Limitations





Will this process eventually solve all forms of bias?

Methodology: Visualization





To recognize **limitations** we should go into the **other direction**.



Methodology:

• Find the most generic definitions for the main approaches in click-based LTR: counterfactual estimation and click modelling.

• From these definitions, **derive** under which **conditions** they cannot be **unbiased** or **consistent**:

$$\underbrace{\forall (d,q), \ \mathbb{E}\big[\hat{R}_{d|q}\big] = R_{d|q}}_{\text{unbiasedness}}, \qquad \qquad \forall (d,q)$$

$$\underbrace{\forall (d,q), \lim_{|\mathcal{D}_q| \to \infty} \hat{R}_{d|q} = R_{d|q}}_{\text{consistency}}.$$

Limitations of Click-Based Counterfactual Estimators



Display Context:

x(d) contains all information about how d is displayed, but no information about relevance $R_{d|q}$.



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Counterfactual Relevance Estimate:

Average over independently-sampled interactions, each transformed by function *f*:

$$\hat{R}_{d|q} = \frac{1}{N} \sum_{i=1}^{N} f(c_i(d), x_i(d)).$$

f only has two relevant values per context x(d): f(1, x(d)) and f(0, x(d)).



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This definition captures virtually all existing counterfactual LTR methods.



Derivation in a **nutshell**:

$$\mathbb{E}_{c,x}\left[\hat{R}_{d|q}\right] = R_{d|q} \longleftrightarrow \lim_{|\mathcal{D}_q| \to \infty} \hat{R}_{d|q} = R_{d|q} \longleftrightarrow \mathbb{E}_{c,x}[f(c(d), x(d))] = R_{d|q},$$



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the **expected value** can be rewritten to:

$$\mathbb{E}_{c,x}[f(c(d), x(d)) \mid q] \\ = \mathbb{E}_x[P(C = 1 \mid d, x, q)f(1, x(d)) + (1 - P(C = 1 \mid d, x, q))f(0, x(d)) \mid q] \\ = \mathbb{E}_x[P(C = 1 \mid d, x, q)(f(1, x(d)) - f(0, x(d))) + f(0, x(d)) \mid q].$$



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Thus unbiasedness or consistency are **only possible** when click probabilities are an **affine transformation** of relevance s.t:

$$R_{d|q} = \mathbb{E}_x \bigg[\frac{P(C=1 \mid d, x, q) - \beta_{x(d)}}{\alpha_{x(d)}} \mid q \bigg].$$

Click-Based Counterfactual Estimators: Derivation 2



From our generic definition, we derived a very broad condition:

$$R_{d|q} = \mathbb{E}_x \left[\frac{P(C=1 \mid d, x, q) - \beta_{x(d)}}{\alpha_{x(d)}} \mid q \right],$$

but it is hard to interpret as it relies on the distribution over both x and c.

Remember: $x \sim \text{logging policy, and } c \sim \text{user behavior given } x$.

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Remember: $x \sim \text{logging policy, and } c \sim \text{user behavior given } x$.

If **assumptions** on the logging policy are **impossible**, or under a **deterministic** policy, we get the simpler condition:

$$P(C=1 \mid d, x, q) = \alpha_{x(d)} R_{d|q} + \beta_{x(d)}.$$

Click-Based Counterfactual Estimators: Visualization

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Intuitive Interpretation:

 unbiasedness and consistency of counterfactual estimation are only possible under a linear relation between click probability and item relevance.

Reason is that click-based counterfactual estimates are **linear interpolations**.



Limitations of Click-Modelling



A click-model is defined by a loss function $\mathcal{L}(\hat{R}_q, \hat{Z}, \mathcal{D}_q)$ with as input:

- estimated **relevance** variables \hat{R}_q ,
- estimated **latent** variables \hat{Z} ,
- observed data \mathcal{D}_q .

The relevance estimate of a click-model are any values that minimize \mathcal{L} :

$$\left(\hat{R}_{q}^{*}, \hat{Z}^{*}\right) = \arg\min_{\hat{R}_{q}, \hat{Z}} \mathcal{L}(\hat{R}_{q}, \hat{Z}, \mathcal{D}_{q}).$$



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Captures both traditional graphical models and recent deep-learning models.



A click modelling method is consistent **iff** the **only relevance estimates** that **minimize** its **loss** are the **true relevances** as $|D_q|$ tends to infinity:

$$\lim_{|\mathcal{D}_q|\to\infty} \left(R_q = \hat{R}_q \longleftrightarrow \min_{\hat{Z}} \mathcal{L}(\hat{R}_q, \hat{Z}, \mathcal{D}_q) = \min_{\hat{R}'_q, \hat{Z}} \mathcal{L}(\hat{R}'_q, \hat{Z}, \mathcal{D}_q) \right).$$



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A click modelling method is unbiased **iff** the **expected value** of its optimal relevance estimates are equal to the **true relevances**:

$$\mathbb{E}_{\mathcal{D}_q} \big[\hat{R}_q^* \big] = R_q \longleftrightarrow \forall d, \ \mathbb{E}_{\mathcal{D}_q} \big[\hat{R}_{d|q}^* \big] = R_{d|q}.$$



Consistency depends **both** on the loss/model **and** how **data** is **gathered**.



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	Ranking 1			Ranking 2				
ltems	A	В	С	D	В	А	D	С
Click Prob.	0.90	0.64	0.40	0.05	0.8	0.72	0.20	0.10

Fitting a rank-based position-bias model to this scenario: $\hat{P}(C = 1 \mid d, k, q) = \hat{\alpha}_k \hat{R}_{d|q}$ with $\hat{\alpha}_1 = 1$



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Problem: Different relevance estimates equally predictive:

•
$$\hat{\alpha}_2 = 0.8$$
, $\hat{\alpha}_3 = 4 \cdot \hat{\alpha}_4$,

•
$$\hat{R}_A = 0.9$$
, $\hat{R}_B = 0.8$ and $\hat{R}_C = 2 \cdot \hat{R}_D$.

Conclusion





Main Findings

• Counterfactual estimation can only be unbiased when clicks follow affine transformations of relevance.



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- Counterfactual estimation can only be unbiased when clicks follow affine transformations of relevance.
- Consistency of **click-modelling** methods depend on their **expected loss minima**; unclear if robust unbiasedness guarantees are possible.
- **"Unbiased LambdaMART"** is **not** a sound debiasing method; illustrative example of why to be careful with assumptions.



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Unbiasedness may **not** always be **possible**:

- we should not invariably expect nor require it of future work;
- unbiasedness may not be a realistic long term goal, field will likely shift to bias mitigation or partial debiasing;
- good reason to re-name the field: Click-Based LTR.





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